King Mongkut's University of Technology Thonburi
Midterm Exam of First Semester, Academic Year 2018

Student Name: ……………………………………………………………………………

Student ID: ………………………………………………Seat No.: ………………..

MTH 303 Numerical Methods (International Program)

Examination Date: 8 October 2018 Time: 9.00 – 12.00

Instructions

1. This examination paper contains 9 pages (including this cover page and the formula).
2. Total score for this examination is **80** marks.
3. The answers must be written in the answer sheet provided.
4. Dictionary is not allowed.
5. Text, Lecture or any other documents cannot be taken into the examination room.
6. The use of a calculator is permitted.

No.	1	2	3	4	5					Total
Score										

Instructor: Dr. Saeid Zahmatkesh

phone: 9549

All questions in this examination have been approved by the Department of Mathematics.

Wiboonsak Watthayu

(Dr. Wiboonsak Watthayu)
Head Department of Mathematics

1. Let x be a number which is stored into a computer with 32 bits in binary system by **IEEE** standard as follows:

1 10001001 10010110001000000000000

Find the value of x in decimal system (5 marks).

2. Let $P(x) = 0.4x^3 + 0.2x^2 + 0.6x - 1$.

   (a) Show that the equation $P(x) = 0$ has exactly one solution in $[0, 1]$ (2 marks).

   (b) Then, apply **Newton's method** with the initial value $x_0 = 0.8$ combining with **Horners's method** to find an approximation $\tilde{x}$ of the solution such that $|P(\tilde{x})| < 0.5 \times 10^{-3}$ (12 marks).

   (c) Round $\tilde{x}$ to 3 decimal places (1 mark).

3. Suppose that $f(x) = x^3e^{-x} + 1$.

(a) Use the graphical method to find an interval $[a, b]$ which contains the solution of the equation $f(x) = 0$, and on which **fixed-point iteration** will converge (5 marks).

(b) Thus, by using the method of **fixed-point iteration**, find an approximation $\tilde{x} = x_n$ of the solution such that $|x_n - x_{n-1}| < 0.5 \times 10^{-3}$ (**Hint:** Use $x_0 = b$ from the interval $[a, b]$ as the initial value) (9 marks).

(c) Round $\tilde{x}$ to 3 decimal places (1 mark).

4. Consider the following data $y_i = f(x_i)$ for $i = 0, 1, 2, 3, 4$:

$f(0) = 2$ $f(0.25) = 2.6487$ $f(0.5) = 3.7183$ $f(0.75) = 4.4817$ $f(1) = 3.6522$

(a) Set up the **Finite Divided Difference** table (8 marks).

(b) Use **Newton's backward difference formula** to estimate f(0.8) (5 marks).

(c) Use **Gauss forward formula** to estimate f(0.6) (5 marks).

(d) Estimate $f(0.15)$ by using **Lagrange interpolating polynomials** of degree 2 and the following data $y_i = f(x_i)$ for $i = 0, 1, 2$:

$$f(0) = 2 \quad f(0.25) = 2.6487 \quad f(0.5) = 3.7183$$

(12 marks)

Attention! In all parts in above, round the answer to 4 decimal places.

5. Apply the Simpson's 1/3 rule to approximate the definite integral

$$\int_0^1 \frac{1}{1+x}\,dx$$

such that $|\text{error}| < 0.5 \times 10^{-4}$. Then, round the answer to 4 decimal places (15 marks).

Method of False Position

$$x = \frac{XL\, f(XR) - XR\, f(XL)}{f(XR) - f(XL)}$$

Bisection Method

$$x = \frac{XL + XR}{2}$$

Secant Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f(x_i) - f(x_{i-1})}(x_i - x_{i-1})$$

Newton's Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad ; i = 0, 1, 2, \ldots$$

Gauss Elimination

$$\left[\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & a_{1n+1} \\ & a_{22}^{(1)} & a_{23}^{(1)} & \cdots & a_{2n}^{(1)} & a_{2n+1}^{(1)} \\ & & a_{33}^{(2)} & \cdots & a_{3n}^{(2)} & a_{3n+1}^{(2)} \\ & & & \ddots & \vdots & \vdots \\ & & & & a_{nn}^{(n-1)} & a_{nn+1}^{(n-1)} \end{array}\right]$$

where $m_{ij} = \dfrac{a_{ij}^{(j-1)}}{a_{jj}^{(j-1)}}$; $a_{jj}^{(j-1)} \neq 0$

$$a_{ik}^{(j)} = a_{ik}^{(j-1)} - m_{ij} a_{jk}^{(j-1)}$$

$j = 1, 2, \ldots, n-1$ (step of elimination)

$i = j+1, j+2, \ldots, n$

$k = j, j+1, \ldots, n+1$

Newton's forward difference formula

$$f(\bar{x}) = y_0 + \binom{s}{1}\Delta y_0 + \binom{s}{2}\Delta^2 y_0 + \ldots + \binom{s}{n}\Delta^n y_0$$

where $s = \dfrac{\bar{x} - x_0}{h}$, $h = x_{i+1} - x_i$

$$\binom{s}{n} = \frac{s(s-1)(s-2)\cdots(s-n+1)}{n!}$$

Newton's backward difference formula

$$f(x)=y_n+\binom{s}{1}\nabla y_n+\binom{s+1}{2}\nabla^2 y_n+\binom{s+2}{3}\nabla^3 y_n$$

$$+\ldots+\binom{s+n-1}{n}\nabla^n y_n$$

where $s=\dfrac{\bar{x}-x_n}{h}$, $h=x_{i+1}-x_i$

$$\binom{s}{n}=\frac{s(s-1)(s-2)\cdots(s-n+1)}{n!}$$

Gauss Interpolation formula

Gauss-forward formula

$$f(\bar{x})=y_0+\binom{s}{1}\delta y_{\frac{1}{2}}+\binom{s}{2}\delta^2 y_0+\binom{s+1}{3}\delta^3 y_{\frac{1}{2}}+\binom{s+1}{4}\delta^4 y_0+\ldots$$

Gauss-backward formula

$$f(\bar{x}) = y_0+\binom{s}{1}\delta y_{-\frac{1}{2}}+\binom{s+1}{2}\delta^2 y_0+\binom{s+1}{3}\delta^3 y_{-\frac{1}{2}}$$

$$+\binom{s+2}{4}\delta^4 y_0+\binom{s+2}{5}\delta^5 y_{-\frac{1}{2}}+\ldots$$

Lagrange Interpolation polynomial degree n

$$P_n(x)=L_0(x)y_0+L_1(x)y_1+\ldots+L_n(x)y_n=\sum_{k=0}^{n}L_k(x)y_k$$

$$L_k(x)=\frac{(x-x_0)(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)}$$

$$=\prod_{\substack{i=0\\ i\neq k}}^{n}\frac{(x-x_i)}{(x_k-x_i)}$$

$$L_k(x_i)=\begin{cases}0, & i\neq k\\ 1, & i=k\end{cases},\quad 0\le i\le n$$

Trapezoidal rule

$$\int_a^b f(x)dx \approx (h/2)[f(x_0) + f(x_n)] + h\sum_{i=1}^{n-1} f(x_i)$$

$$h = (b-a)/n$$

$$x_0 = a, x_n = b$$

$$x_i = a + ih; \quad i = 0, 1, 2, ..., n$$

$$|\text{Error}| \leq (|b-a|/12)h^2 \max_{a\leq x\leq b} |f^{(2)}(x)|$$

Simpson's 1/3 rule

$$\int_a^b f(x)dx \approx (h/3)[f(a) + f(b) + 4\sum_{i=1}^{m} f(x_{2i-1}) + 2\sum_{i=1}^{m-1} f(x_{2i})]$$

$$n = 2m$$

$$h = (b-a)/n$$

$$x_0 = a, x_n = b$$

$$x_i = a + ih; \quad i = 0, 1, 2, ..., n$$

$$|\text{Error}| \leq (|b-a|/180)h^4 \max_{a\leq x\leq b} |f^{(4)}(x)|$$