14 ต.ค.61

Seat No.

King Mongkut's University of Technology Thonburi

Midterm Examination, Semester 1/2018

Subject: MTH101 Mathematics I (International program) For: First Year Engineering Students

Date: Monday 1st October 2018 Time: 9.00-12.00

Instructions:

1. This exam contains 12 pages including this cover sheet and a formula sheet.
2. This exam consists of 11 problems (6 problems in part A and 5 problems in part B) whose score are added up to 80 points in total. (41 points in part A and 39 points in part B)
3. Neither a dictionary nor a calculator is allowed.
4. Any documents and textbooks are not permitted.
5. Write down your name and ID on every page before working on the exam.
6. Show your work and answer all questions neatly in the exam paper.
7. Writing with a dark pencil is acceptable.
8. Read each problem very carefully before answering.

Dr. Pawaton Kaemawichanurat
Dr. Anuwat Tangthanawatsakul
Examination Writers.

Name.. I.D.................................... Department..........

This examination paper had been approved by the committee of the department of mathematics.

Wiboonsak Watthayu

(Dr. Wiboonsak Watthayu)

Head of Mathematics Department

Name .... I.D. .... Department ....

Part A (41 Points)

1. (Total 13 Points) Find the limits of the following functions.

1.1) (4 points) $\lim\limits_{x\to 0}\dfrac{1-\cos^2 x}{\sqrt{1+\sin^2 x}-1}$

1.2) (4 points) $\lim\limits_{y\to\infty}\tan\left(\dfrac{y\pi+4}{4y+\sqrt{y}}\right)$

Name .... I.D. .... Department ....

1.3) (5 points) $\lim_{x \to 0} \left(\frac{1}{\ln(x+1)} - \frac{1}{\sin x} \right)$

Name .... I.D. .... Department ....

2. (7 points) Let f be the function defined by

$$f(x)=\begin{cases} \dfrac{\sin x}{\sqrt{Bx^2+x^3}} & ; x<0 \\ A & ; x=0 \\ \dfrac{e^x-e^{3x}}{(1-x)\ln(x+1)} & ; x>0. \end{cases}$$

Find the constants A and B so that this function is continuous at $x=0$.

Name .... I.D: .... Department ....

3. (5 points) Find $\frac{d}{dx}\cos x$ by using the definition of derivative.

4. (4 points) In Physics Laboratory, a particle is moving along the X-axis. Its distance from the sensor is given by

$$s(t) = 2\cos\left(\frac{\pi}{2}t\right) - \sin\left(\frac{\pi}{4}t\right).$$

Find the average velocity of this particle from $t = 2$ to $t = 6$ and find the instantaneous rate of change of s at $t=6$.

5. (5 points) Find $\dfrac{d}{dx}\left(e^{\frac{\arctan x}{1+x^2}}\right)$.

6. (7 points) Find $\dfrac{dy}{dx}$ where y is a function of x and is implicitly defined as

$$x^{\log_3 y} - \left(\sqrt{\frac{y+1}{y+3}}\right)^x = 0$$

Name .... I.D. .... Department ....

Part B (39 Points)

7. (4 points) Let f be the function defined by $f(x)=\dfrac{1}{\sqrt{1-x}}$ for all real numbers $x<1$.

   Find $f^{(2561)}(x)$.

8. (5 Points) Find the values of the constants a, b and c such that the graph of $g(x)=ax^3+bx^2+cx$ passes through the point $(-1,0)$ and has an inflection point at the point $(1,1)$.

Name .... I.D: .... Department ....

9. (Total 17 Points) Let $f(x)=\dfrac{e^x}{x-1}$ for any real numbers $x \neq 1$.

9.1) (3 Points) Find all horizontal and vertical asymptotes of the function f.

9.2) (3 Points) Find the intervals on which f is increasing and which f is decreasing.

Name .... I.D: .... Department ....

9.3) (3 Points) Find all critical points of the function f and determine if each critical point gives a relative maximum, a relative minimum or neither.

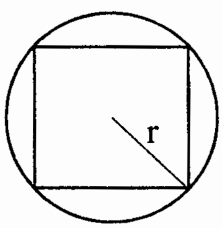
9.4) (3 Points) Find the intervals on which f is concave up and which f is concave down. (Hint: Note that $x^2 - 4x + 5 = x^2 - 4x + 4 + 1 = (x-2)^2 + 1 > 0$ for all $x \in \mathbb{R}$.)

9.5) (2 Points) Find all inflection points of the function f.

Name .... I.D. :.. Department ....

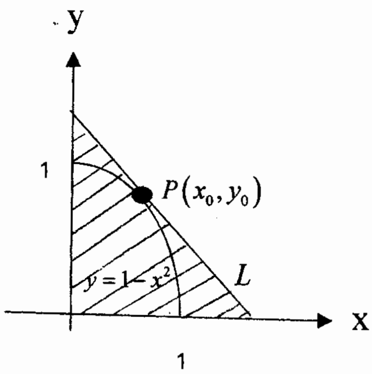
9.6) (3 Points) Sketch a graph of the function f.

10. (6 Points) A square is inscribed in a circle of radius r as shown in the figure below. Assume that the radius of this circle is increasing at a rate of 4 inches/second. Find the rate of change of the area of the square when the (instant) radius of this circle equals to 2 inches.



Name .... I.D: .... Department ....

11. (7 Points) Find the equation of the tangent line L to the graph of the parabola $y = 1 - x^2$ at the point $P(x_0, y_0)$ such that the triangle in the first quadrant enclosed by the x-axis, the y-axis and the line L has minimum area.



DERIVATIVES FORMULAS

1. $\frac{d}{dx}(c) = 0$, c is a constant
2. $\frac{d}{dx}(cu) = c\frac{du}{dx}$, $u = u(x)$
3. $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$
4. $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$
5. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
6. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$, $v \neq 0$
7. $\frac{d}{dx}(\sin u) = \cos u\frac{du}{dx}$
8. $\frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$
9. $\frac{d}{dx}(\tan u) = \sec^2 u\frac{du}{dx}$
10. $\frac{d}{dx}(\cot u) = -\csc^2 u\frac{du}{dx}$
11. $\frac{d}{dx}(\sec u) = \sec u \tan u\frac{du}{dx}$
12. $\frac{d}{dx}(\csc u) = -\csc u \cot u\frac{du}{dx}$
13. $\frac{d}{dx}(\arcsin u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$, $-1 < u < 1$
14. $\frac{d}{dx}(\arccos u) = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$, $-1 < u < 1$
15. $\frac{d}{dx}(\arctan u) = \frac{1}{1+u^2}\frac{du}{dx}$
16. $\frac{d}{dx}(\text{arccot}\, u) = -\frac{1}{1+u^2}\frac{du}{dx}$
17. $\frac{d}{dx}(\text{arcsec}\, u) = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$, $|u| > 1$
18. $\frac{d}{dx}(\text{arccsc}\, u) = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$, $|u| > 1$
19. $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$
20. $\frac{d}{dx}(\log_a u) = \frac{1}{u \ln a}\frac{du}{dx}$, $a \neq 0, 1$
21. $\frac{d}{dx}e^u = e^u\frac{du}{dx}$
22. $\frac{d}{dx}a^u = a^u \ln a\frac{du}{dx}$

Trigonometric Formulas

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sin^2 A + \cos^2 A = 1$

$\sec^2 A - \tan^2 A = 1$

$\csc^2 A - \cot^2 A = 1$

$\sin^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2}\cos 2A$