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Seat Number

King Mongkut's University of Technology Thonburi
Midterm Examination
Semester 1 -- Academic Year 2017

Subject: EIE 301 Introduction to Probability and Random Processes for Engineers
For: Electrical Communication and Electronic Engineering, 3rd Yr (Inter. Program)
Exam Date: Tuesday October 3, 2017 **Time:** 9.00am-12.00pm

Instructions:-

1. This exam consists of 5 problems with a total of 13 pages, including the cover.
2. This exam is closed books.
3. You are **not** allowed to use a written A4 note for this exam.
4. Answer each problem on the exam itself.
5. A calculator compiling with the university rule is allowed.
6. A dictionary is **not** allowed.
7. **Do not** bring any exam papers and answer sheets outside the exam room.
8. Open Minds … No Cheating! GOOD LUCK!!!

Remarks:-

- **Raise your hand when you finish the exam to ask for a permission to leave the exam room.**
- **Students who fail to follow the exam instruction might eventually result in a failure of the class or may receive the highest punishment within university rules.**
- **Carefully read the entire exam before you start to solve problems. Before jumping into the mathematics, think about what the question is asking. Investing a few minutes of thought may allow you to avoid twenty minutes of needless calculation!**

Question No.	1	2	3	4	5	TOTAL
Full Score	20	20	20	20	20	100
Graded Score						

Name ______________________________ **Student ID** ________________

This examination is designed by
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This examination has been approved by the committees of the ENE department.

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Problem 1: Fill in the Blanks [20 points]

This problem consists of unrelated parts, which can be answered separately. Fill in the blanks for each question. You do not need to justify your answers.

Part 1: Data Analysis

A marketer wants to understand the number of mobile phones that each Thai citizen owns. The marketer surveys a sample of 1,000 Thai citizens and collects the following data.

Number of mobile phones owned	Number of Thai citizens	Relative frequency
0	40	________
1	800	________
2	100	________
3	60	________
Sum	1,000	________

(a) [2.5 points] Fill out the relative frequencies in the above table.

(b) [2.5 points] Find the sample proportion of Thai citizens who own two or more mobile phones.

Answer = ______________________________

Part 2: Random Experiment

A student rolls two dice together and record the faces that show up. Suppose that each dice consists of three faces: A, B, and C. Furthermore, suppose that the two dice are identical, so that the order of their faces in an outcome is unimportant.

(a) [2.5 points] Identify the sample space.

Ω = ______________________________

(b) [2.5 points] Let E denote an event that none of the two dice rolls to face A. Identify all outcomes in event E.

E = ______________________________

Part 3: Souvenir Shop

A souvenir shop sells two types of products: postcards and key chains. Of all customers who enter the souvenir shop, 20% purchase nothing, 30% purchase postcards, and 60% purchase key chains.

(c) [5 points] Suppose that a customer enters the souvenir shop. Find the probability that the customer will purchase a postcard and a key chain.

Answer = ______________________________

(d) [5 points] Given that a randomly-selected customer does not purchase a key chain, find the conditional probability that the customer purchases a postcard.

Answer = ______________________________

Problem 2: Data Transformation [20 points]

A sample $x_1, x_2, \ldots, x_{10}$ of size ten is transformed into another sample $y_1, y_2, \ldots, y_{10}$, where

$$y_i = -3x_i^2 + 4, \qquad i = 1, 2, 3, \ldots .$$

The sample mean and sample standard deviation of $x_1, x_2, \ldots, x_{10}$ are

$$\bar{x} = -2, \qquad s = 8,$$

respectively.

(a) [10 points] Find the sample mean of $y_1, y_2, \ldots, y_{10}$.

(b) [10 points] Suppose that an additional measurement x_{11} is available where $x_{11} = 0$. Find the sample mean of $y_1, y_2, \ldots, y_{10}, y_{11}$.

Problem 3: Music Playlist [20 points]

A playlist consists of 2 Japanese songs, 3 English songs, and 5 Thai songs. Suppose that songs in the playlist are shuffled in a random order.

(a) [10 points] Find the probability that the two Japanese songs do not appear next to each other in the playlist.

(b) [10 points] Find the probability that a Thai song is first <u>or</u> second in the playlist.

Problem 4: Library Service [20 points]

A library opens daily from 6am to 6pm. Historical data indicate that 20% of library visitors arrive in the morning (6am–10am), 30% arrive at midday (10:01am–2pm), and 50% arrive in the afternoon (2:01pm–6pm). In addition, 70% of all morning visitors check out (borrow) library books, while 40% of the midday visitors and 60% of the afternoon visitors check out library books.

Suppose a visitor is randomly chosen.

(a) [10 points] If this visitor checked out a library book, what is the probability that the visitor arrived in the morning?

(b) [10 points] If this visitor did not check out a library book, what is the probability that the visitor arrived in the afternoon?

Problem 5: Coin Tosses [20 points]

A coin has the probability of 0.7 to turn head. This coin is tossed five times in a row. Assume that successive tosses are independent.

(a) [10 points] Find the probability that the 1st toss turns head and the 2nd toss turns tail.

(b) [10 points] Find the probability that one of the five tosses turns head and the remaining four tosses turn tail.

Formula Sheet

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\mu = \frac{x_1 + x_2 + \cdots + x_N}{N} = \frac{\sum_{i=1}^{N} x_i}{N}$$

$$\tilde{x} = \begin{cases} x'_m & \text{if } n \text{ is odd, where } m = \frac{n+1}{2} \\ \frac{1}{2}(x'_m + x'_{m+1}) & \text{if } n \text{ is even, where } m = \frac{n}{2} \end{cases}$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}\left[\left(\sum_{i=1}^{n} x_i^2\right) - n(\bar{x})^2\right]$$

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2 = \frac{1}{N}\left(\sum_{i=1}^{N} x_i^2\right) - \mu^2$$

For $y_i = ax_i + b$, where $i = 1, 2, \ldots, n$,

$$\bar{y} = a\bar{x} + b, \quad s_y^2 = a^2 s_x^2, \quad s_y = |a| s_x$$

Axioms of probability:

1. The probability of any event A is non-negative: $\mathbb{P}\{A\} \geq 0$
2. The probability of the sample space equals one: $\mathbb{P}\{\Omega\} = 1$
3. If $A_1, A_2, A_3, \ldots$ are pairwise disjoint events, then $\mathbb{P}\{\cup_{i=1}^{\infty} A_i\} = \sum_{i=1}^{\infty} \mathbb{P}\{A_i\}$

- The probability of the null event equals zero: $\mathbb{P}\{\emptyset\} = 0$
- If events $A_1, A_2, \ldots, A_n$ are pairwise disjoint, then $\mathbb{P}\{\cup_{i=1}^{n} A_i\} = \sum_{i=1}^{n} \mathbb{P}\{A_i\}$
- For any event A, $\mathbb{P}\{A'\} = 1 - \mathbb{P}\{A\}$
- The probability of any event A is at most one: $\mathbb{P}\{A\} \leq 1$
- For any two events A and B, $\mathbb{P}\{A \cup B\} = \mathbb{P}\{A\} + \mathbb{P}\{B\} - \mathbb{P}\{A \cap B\}$
- For any three events A, B, and C,

$$\begin{aligned}\mathbb{P}\{A \cup B \cup C\} &= \mathbb{P}\{A\} + \mathbb{P}\{B\} + \mathbb{P}\{C\} \\ &\quad - \mathbb{P}\{A \cap B\} - \mathbb{P}\{A \cap C\} - \mathbb{P}\{B \cap C\} + \mathbb{P}\{A \cap B \cap C\}\end{aligned}$$

- When outcomes are equally likely, $\mathbb{P}\{A\} = \frac{|A|}{|\Omega|}$
- *The product rule*: the number of k-tuples is $n_1 n_2 n_3 \cdots n_k$

$$^{n}P_k = n \times (n-1) \times (n-2) \times \cdots \times (n-k+1) = \frac{n!}{(n-k)!}$$

$$0! = 1, \quad n! = n(n-1)(n-2)\cdots 1$$

$$\binom{n}{k} = \frac{^{n}P_k}{k!} = \frac{n!}{k!(n-k)!}$$

$$\mathbb{P}\{A \mid B\} = \frac{\mathbb{P}\{A \cap B\}}{\mathbb{P}\{B\}}$$

$$\mathbb{P}\{A \cap B\} = \mathbb{P}\{A\}\mathbb{P}\{B \mid A\} = \mathbb{P}\{B\}\mathbb{P}\{A \mid B\}$$

$$\mathbb{P}\{A_1 \cap A_2 \cap A_3\} = \mathbb{P}\{A_3 \mid A_2 \cap A_1\} \cdot \mathbb{P}\{A_2 \mid A_1\} \cdot \mathbb{P}\{A_1\}$$

The law of total probability: Suppose events $A_1, A_2, \ldots, A_n$ partition the sample space Ω. Then the probability of any other event B equals

$$\mathbb{P}\{B\} = \sum_{i=1}^{n} \mathbb{P}\{B \mid A_i\}\mathbb{P}\{A_i\}$$

Bayes' theorem: : Suppose events $A_1, A_2, \ldots, A_n$ partition the sample space Ω and have the probabilities $\mathbb{P}\{A_1\}, \mathbb{P}\{A_2\}, \ldots, \mathbb{P}\{A_n\}$. Let B denote any event that has a chance to occur, i.e., $\mathbb{P}\{B\} > 0$. Then the posterior probability of A_j given that B has occurred is

$$\mathbb{P}\{A_j \mid B\} = \frac{\mathbb{P}\{A_j \cap B\}}{\mathbb{P}\{B\}} = \frac{\mathbb{P}\{B \mid A_j\}\mathbb{P}\{A_j\}}{\sum_{i=1}^{n} \mathbb{P}\{B \mid A_i\}\mathbb{P}\{A_i\}}$$

for each index $j = 1, 2, \ldots, n$.

- Events A and B are independent $\iff \mathbb{P}\{A \mid B\} = \mathbb{P}\{A\}$ $\iff \mathbb{P}\{B \mid A\} = \mathbb{P}\{B\} \iff \mathbb{P}\{A \cap B\} = \mathbb{P}\{A\}\mathbb{P}\{B\}$
- Events $A_1, A_2, \ldots, A_n$ are mutually independent iff for every size $k \in \{2, 3, \ldots, n\}$ and for every subset of indices $i_1, i_2, \ldots, i_k$, the probability of an intersection equals the product of probabilities:

$$\mathbb{P}\{A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}\} = \mathbb{P}\{A_{i_1}\} \cdot \mathbb{P}\{A_{i_2}\} \cdots \mathbb{P}\{A_{i_k}\}$$