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King Mongkut's University of Technology Thonburi

Seat No.

Midterm Examination, 1/2017

Subject: EIE 208 Electrical Engineering Mathematics

For: 2nd yr. students, Dept. of Electronic and Telecommunication Engineering

Date: Thur., Septempber 28, 2017 1:00pm – 4:00pm

Instructions:-

1. This exam consists of 10 pages (including this page) for 7 problems with the total score of 90.
2. This exam is *closed books*. Textbooks and documents related to the subject are not allowed.
3. *Answer* each problem on the exam itself (use the back pages for extra spaces).
4. A calculator complying the university rules is allowed.
5. *Dictionaries* are not allowed.
6. Do not bring any exam papers and answer sheets outside the exam room.

Remarks:-

- Raise your hand when you finish the exam to ask for a permission to leave the exam room.
- Students who fail to follow the exam instruction might eventually result in a failure of the class or may receive the highest punishment with university rules.
- Carefully read the entire exam before you start to solve problems. Before jumping into the mathematics, think about what the question is asking. Investing a few minutes of thought may allow you to avoid twenty minutes of needless calculation!

Open Minds ... No Cheating! GOOD LUCK!!!

This exam is designed by Asst. Prof. Dr. Pinit Kumhom (Ext. 9075, 9070)

This examination has been approved by the committees of the ENE department.

R. Silapunt

(Assoc. Prof. Dr. Rardchawadee Silapunt)
Head of Electronic and Telecommunication Engineering Department

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Name-Surname: Student No.:

Prob. No.	1	2	3	4	5	6	7	8	9	10	Total
Full Score	10	15	10	12	15	8	20				90
Recieved Score											

Problem 1 [Math. and Engineering] (10 points) Describe the importance of mathematics in engineering and how mathematics is applied to engineering.

Problem 2 [Signals] (15 points)

2.1 (5 points) Each of the following statements contains some part (parts) that is (are) not correct. Rewriting them so that they are correct.

1. A signal is a time function whose domain and co-domain are sets of numbers representing a time duration and signal's values, respectively. While both domain and co-domain of continuous signal are sets of real numbers, a domain of a discrete signal is a set of real numbers and its co-domain is a set of Integers.

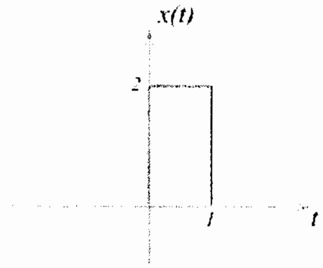
2. A sinusoidal signal can be completely specified by its amplitude and frequency.

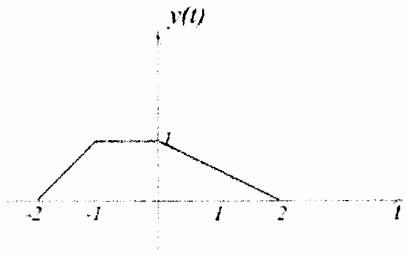
3. If $x(t)$ is a periodic continuous signal, then $x(t) = x(t + kf)$, where f is a positive real number representing the frequency of the signal, and $k = 0, \pm 1, \pm 2, \ldots$

4. A complex signal $z(t) = z_0 + re^{j2\pi t}, 0 \leq t \leq 2$ plots complex points on the circle with radius r and the center at the point 0 for one cycle.

5. A signal is said to be of a *finite support* if it is zero only over finite time interval.

2.2 (10 points) Given a signal $x(t)$ and $y(t)$ shown below, plot $x(-0.5t-1)$, $y(2t+1)$ and $x(t)y(-t-1)$





Problem 3 [Signals] (10 points) Sketch the trajectory of the complex function $z(t) = 1 - i + e^{-i\pi/4}e^{i100\pi t}$ for $0 \leq t < 20ms$ on the z-plane, and sketch its real and imaginary parts as a function of t.

Problem 4 [Complex numbers] (12 points) Manipulate the following complex numbers so that they can be expressed in (1) rectangular form, $z = x + iy$, and (2) polar form using exponential function, $z = re^{i\theta}$, where θ is the argument of z in the principal branch. Show how your results are obtained.

4.1) $z = \frac{1}{2}(\sqrt{3}+i)e^{i\frac{\pi}{2}}+2e^{i\frac{2\pi}{3}}$

4.2) $z = (1-\pi)e^{\pi}e^{i\frac{\pi}{6}} - \frac{(1-\pi)e^{-i\frac{\pi}{6}}}{e^{-\pi}}$

4.3) $z = \frac{2e^{-i\frac{\pi}{6}}(1+\sqrt{3}i)}{e^{i\frac{\pi}{3}}} - \frac{\frac{1}{2}(1-i\sqrt{3})}{e^{-i\frac{5\pi}{6}}}$

Problem 5 [Complex Roots] (14 points) Find all the solutions of the following equation and sketch them on the complex plane.

Hint: Use the quadractic equation, and for a complex constant $c = re^{i\theta}$

$$z_k = \sqrt[n]{r}e^{i\theta/n}e^{i2\pi k/n} \quad (k = 0, 1, \ldots, n-1)$$

where $z_k, k = 0, 1, 2, \ldots, n-1$ are all the n^{th} root of c.

$$(z^3+8)(z^2+2e^{i\frac{\pi}{6}}z+e^{i\frac{\pi}{3}}) = 0$$

Problem 6 [Analytic Functions] (8 points) Find the domain that make the following complex function analytic. (Hint: Use the Cauchy-Riemann Equations, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$)

6.1 $f(z) = 2x^2 - 2y^2 + 2y - 2x + i(-2x + 4xy - 2y)$

6.2 $f(z) = e^{x^2}\cos(2y) + ie^{x^2}\sin(2y)$

Problem 7.1 [Rational Function] (10 points) Express the rational function

$$R(z) = \frac{z-2}{(z-1)^2(z^2+4)}$$

in partial fraction form using the following theorem.

Theorem (Partial Fraction)

If

$$R_{m,n}(z) = \frac{p_m(z)}{q_n(z)} = \frac{a_0 + a_1 z + a_2 z^2 + \cdots + a_m z^m}{b_n(z-c_1)^{d_1}(z-c_2)^{d_2}\cdots(z-c_r)^{d_r}} \tag{1}$$

is a rational function whose *denominator degree* $n = d_1 + d_2 + \cdots + d_r$ *exceeds* its *numerator degree* m, then $R_{m,n}$ has a *partial decomposition* of the form

$$\begin{aligned} R_{m,n}(z) &= \frac{A_0^{(1)}}{(z-c_1)^{d_1}} + \frac{A_1^{(1)}}{(z-c_1)^{d_1-1}} + \cdots + \frac{A_{d_1-1}^{(1)}}{(z-c_1)} \\ &\quad + \frac{A_0^{(2)}}{(z-c_2)^{d_1}} + \frac{A_1^{(2)}}{(z-c_2)^{d_2-1}} + \cdots + \frac{A_{d_2-1}^{(2)}}{(z-c_2)} \\ &\quad + \cdots + \frac{A_0^{(r)}}{(z-c_r)^{d_r}} + \cdots + \frac{A_{d_r-1}^{(r)}}{(z-c_1)} \end{aligned} \tag{2}$$

where $\{A_s^{(j)}\}$ are constants (The c_k's are assumed *distinct*), and for each c_j with multiplicity d_j,

$$A_s^{(j)} = \lim_{z \to c_j} \frac{1}{s!} \frac{d^s}{dz^s} \left[(z-c_j)^{d_j} R_{m,n}(z) \right] \quad (s = 0, 1, \ldots d_j - 1) \tag{3}$$

Problem 7.2 [Complex Integration] (10 points) Find the integeral $\int_{\Gamma_1} f(z)dz$ and $\int_{\Gamma_2} f(z)dz$, where $f(z)$ is the function $R(z)$ in **Problem 7.1** along the contour Γ_1 and Γ_2 shown in the Figure below.
(Hint: (1) Use the Residue theorem.

(2) Residue of $R(z)$ at its pole z_j with multiplicity d_j is equal to the term $A^{(j)}_{d_j-1}$ of the equation (2) and (3) in **Problem 7.1**)